Self numbers and numbers with 3 and 4 generators

(Volume II of Puzzles of self numbers)

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with best complements
from D. R.
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To Dr.
B. S. Madhav
Rao.

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PREFACE

I have great pleasure in presenting this new book to the reader. This is really the 2nd part of my book puzzles of the self numbers. I hope reader will enjoy the constructions of huge numbers as described here at pages 19–20.

Self number is my new idea and I got this idea while thinking on some problems of Demlo numbers.

In my previous part of the book I have given a table of self numbers from 1 to 1313 and a table of 2 generator numbers and their generators between 1010 to 1015.

In this book I have prepared a list of all two generator numbers till 10000 ; Also I have given some peculiar types of self numbers in the appendix. It is a matter of great astonishment to know that though there are so many self numbers, numbers of 1 generator and numbers of 2 generators further till 10^{13}. The first number with 3 generators has 14 digits and it is $10^{13} + 1$; and that the first number with 4 generators's has 25 digits. Several huge numbers having as many as 400 digits are described in this book and we find great pleasure in preparing such huge numbers.

The invention of self number is only from about 1947 and I read the first paper on this subject at the Bangalore Mathematical Conference in 1952.

I was constructing various types of sets of numbers, for so many years, but the idea of preparing a list of all numbers with 3 and 4 generators and publishing a book for it, came to me only when I read my paper on construction of huge 3–4 generator numbers at the Bharat Ganit Parishad Lucknow in March 1961. A very lively and thought

provoking discussion followed in the meeting as soon as I finished my paper there.

I am thankful to Dr. Ram Vallabah of the Lucknow University Professor of Mathematics for the chance given to me by him for reading my paper there at such a fine meeting of members of the Parishad.

I express my thanks to Prof. K. R. Gunjikar of Bombay for taking interest in my work and giving me some suggestive hints.

I also express my thanks to Professor V. S. Huzurbazzar of the Poona University for his prompt help and guidance at some points and lastly it is declared that the author acknowledges his indebtedness to the University of Poona for the grant-in-aid received by him from the University towards the cost of publication of his book.

I hope reader will surely welcome this Volume II of my book.

All suggestions for the improvement of this subject will be considered favourably. With best regards to all lovers of **number and its properties.**

I declare this book open today.

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N. B.—See please the list of my other publications on the cover of the book.

CONTENTS

Self numbers and numbers with 2, 3 and 4 generators

(Puzzles of the self numbers Volume II)

Definitions

(1) If a number N_2 is such that it is equal to $N_1 + d_1$ where N_1 is some other number and d_1 is the sum of the digits in N_1 then N_2 is the generated number N_1 is the generator of N_2.

(2) If N_2 is such that it cannot be equal to $N_1 + d_1$ then N_2 is a self number.

Explanation

Generator of a number and generated number :— If N is a number and the sum of all the digits in it is d_1 then $N_1 + d_1$ is a new number and will be called generated number from N_1 Let it be called N_2 then $N_2 = N_1 + d_1$ N_2 is generated from N_1. N_1 is the generator.

Thus take $723 = N_1 \quad d_1 = 7 + 2 + 3 = 12$

$$N_2 = 723 + 12 = 735$$

723 is the generator of 735 A series of generated numbers can be prepared in this way.

Can every number have generator ? no, many numbers have no generators. Thus if we take 35, 35 is generated from 31 ($31 + 3 + 1 = 35$) but 31 itself has no generator. It is called a self number 1, 3, 5, 7, 9, 20, 31, 42. These are first self numbers also 1952, 400, 1111 etc. are big self numbers. Every number ~~must~~ have a generator and if it has not, then the number is a self number.

Thus generator of 38 is 28

generator of 100 is 86

While 108, 110, 1300, 1000000 are self numbers for details over this work see my book " Puzzles of self numbers ".

CHAPTER I

The definitions of self number, generated numbers and generators are already given in my previous work published in 1959.

A table of self numbers between 1 to 1289 is given there at page 15. A list of 2 generators is also given at pages 16–17. It is repeated here in a new form for its frequence use in future for preparing new numbers.

There are 81 numbers of 3 digits which have 2 generators.

The first 2 lines in the following series are for the two generators and third item in the same column is the generated number.

Set No.	81 generated numbers with 2 generators								
	91	92	93	94	95	96	97	98	99
1	100	101	102	103	104	105	106	107	108
	101	**103**	**105**	**107**	**109**	**111**	**113**	**115**	**117**
	191	192	193	194	195	196	197	198	199
2	200	201	202	203	204	205	206	207	208
	202	**204**	**206**	**208**	**210**	**212**	**214**	**216**	**218**
	291	292	293	294	295	296	297	298	299
3	300	301	302	303	304	305	306	307	308
	303	**305**	**307**	**309**	**311**	**313**	**315**	**317**	**319**

81 generated numbers with 2 generators

Set No.									
	391	392	393	394	395	396	397	398	399
4	400	401	402	403	404	405	406	407	408
	404	**406**	**408**	**410**	**412**	**414**	**416**	**418**	**420**
	491	492	493	494	495	496	497	498	499
5	500	501	502	503	504	505	506	507	508
	505	**507**	**509**	**511**	**513**	**515**	**517**	**519**	**521**
	591	592	593	594	595	596	597	598	599
6	600	601	602	603	604	605	606	607	608
	606	**608**	**610**	**612**	**614**	**616**	**618**	**620**	**622**
	691	692	693	694	695	696	697	698	699
7	700	701	702	703	704	705	706	707	708
	707	**709**	**711**	**713**	**715**	**717**	**719**	**721**	**723**
	791	792	793	794	795	796	797	798	799
8	800	801	802	803	804	805	806	807	808
	808	**810**	**812**	**814**	**816**	**818**	**820**	**822**	**824**
	891	892	893	894	895	896	897	898	899
9	900	901	902	903	904	905	906	907	908
	909	**911**	**913**	**915**	**917**	**919**	**921**	**923**	**925**

Up till now in 9 separate sections the 81 numbers with their 2 generators are given.

These all numbers have such types of cogenerators that they differ by 9 only. In every section there is a difference of nine only in the first two columns, and the generated number contains only 3 digits.

925 is our heighest 3 digited number of 2 generators. 101 is the lowest number of 3 digits having 2 generators.

If we continue the same line to prepare the next line of generators we will find that we fail to proceed further in the same type of procedure.

If we make the next section as

991	992	993	994
1000	1001	1002	1003

etc. The new number generated is not the same.

Thus 991 gives $991 + 19 = 1010$

While 1000 gives $1000 + 1 = 1001$

and so the next line fails. This failure is due to the fact that further generated numbers of 2 generators are of 4 digits.

Up till now the difference in the 2 generators was of 9. Now the difference is to be kept as 18 and then the line will continue. Thus :—

$$\left.\begin{matrix} 982 \\ 1000 \end{matrix}\right\} \text{both give } 1001 \qquad \left.\begin{matrix} 983 \\ 1001 \end{matrix}\right\} \text{both give } 1003$$

$$1000 - 982 = 18 \qquad 1001 - 983 = 18.$$

The new further lines will contain the difference in first 2 columns as 18 and not 9.

CHAPTER II

Sets of 4 digited numbers having 2 generators

In each set there are 16 numbers. The first two rows are the generators and the 3rd row in every set is the generated number. Here the difference between the 2 cogenerators is 18 and not 9.

	0982	0983	0984	0985	0986	0987	0988	0989
1	1000	1001	1002	1003	1004	1005	1006	1007
	1001	**1003**	**1005**	**1007**	**1009**	**1011**	**1013**	**1015**

0992	0993	0994	0995	0996	0997	0998	0999
1010	1011	1012	1013	1014	1015	1016	1017
1012	**1014**	**1016**	**1018**	**1020**	**1022**	**1024**	**1026**

Observation— In this set of 16 numbers a certain order is seen and in the generated row (3rd row) we do not see 1000 1002 1004 1006 1008 1010 and 1017, 1019, 1021, 1023, 1025 because instead of 2 generators they have one generator and those who have not 1 generator even are self. The following table is given.

Number	1000	1002	1004	1006	1008	1010
Generator (one)	977	978	979	**self**	990	991
Number	1017	1019	1021	1023	1025	
Generator	1008	1009	**self**	1020	1021	

Also note that from the 1st set of 16 numbers. We see consecutive numbers that means differing by one only which have 2 generators.

This is the group 1011 to 1016 and the first such group. So we note it as

1st set of consecutive numbers having 2 generators to each as follows :

1011, 1012, 1013, 1014, 1015 and 1016

The two generators of each of them can be seen from the table given.

Now note all such points in the further sets of numbers

2nd set of 16 numbers.

Set No.								
	1982	1983	1984	1985	1986	1987	1988	1989
2	2000	2001	2002	2003	2004	2005	2006	2007
	2002	**2004**	**2006**	**2008**	**2010**	**2012**	**2014**	**2016**
	1992	1993	1994	1995	1996	1997	1998	1999
	2010	2011	2012	2013	2014	2015	2016	2017
	2013	**2015**	**2017**	**2019**	**2021**	**2023**	**2025**	**2027**

now I need not give here some other details as I had given in the first set. They can be easily calculated. I only give each time the consecutive set of numbers having 2 generators. Such numbers in this set are

2012,, 2013, 2014, 2015, 2016, 2017

Now further I will give only complete sets and the set of consecutive members having 2 generators and they will be called as two generator consecutive set in that section.

3rd set

	2982	2983	2984	2985	2986	2987	2988	2989
3	3000	3001	3002	3003	3004	3005	3006	3007
	3003	**3005**	**3007**	**3009**	**3011**	**3013**	**3015**	**3017**

3rd set

2992	2993	2994	2995	2996	2997	2998	2999
3010	3011	3012	3013	3014	3015	3016	3017
3014	**3016**	**3018**	**3020**	**3022**	**3024**	**3026**	**3028**

The two generator consecutive number set (called hence forward as T. G. C. Set) of the section is

3013, 3014, 3015, 3016, 3017 and 3018

4th set

	3982	3983	3984	3985	3986	3987	3988	3989
4	4000	4001	4002	4003	4004	4005	4006	4007
	4004	**4006**	**4008**	**4010**	**4012**	**4014**	**4016**	**4018**

3992	3993	3994	3995	3996	3997	3998	3999
4010	4011	4012	4013	4014	4015	4016	4017
4015	**4017**	**4019**	**4021**	**4023**	**4025**	**4027**	**4029**

T. G. C. set here is

4014, 4015, 4016, 4017, 4018 and 4019

5th set

	4982	4983	4984	4985	4986	4987	4988	4989
5	5000	5001	5002	5003	5004	5005	5006	5007
	5005	**5007**	**5009**	**5011**	**5013**	**5015**	**5017**	**5019**

4992	4993	4994	4995	4996	4997	4998	4999
5010	5011	5012	5013	5014	5015	5016	5017
5016	**5018**	**5020**	**5022**	**5024**	**5026**	**5028**	**5030**

T. G. C. set in this section is
5015, 5016, 5017, 5018, 5019 and 5020

6th set

	5982	5983	5984	5985	5986	5987	5988	5989
6	6000	6001	6002	6003	6004	6005	6006	6007
	6006	**6008**	**6010**	**6012**	**6014**	**6016**	**6018**	**6020**
	5992	5993	5994	5995	5996	5997	5998	6999
	6010	6011	6012	6013	6014	6015	6016	6017
	6017	**6019**	**6021**	**6023**	**6025**	**6027**	**6029**	**6031**

T. G. C. set of this section is
6016, 6017, 6018, 6019, 6020 and 6021.

7th set

	6982	6983	6984	6985	6986	6987	6988	6989
7	7000	7001	7002	7003	7004	7005	7006	7007
	7007	**7009**	**7011**	**1013**	**7015**	**7017**	**7019**	**7021**
	6992	6993	6994	6995	6996	6997	6998	6999
	7010	7011	7012	7013	7014	7015	7016	7017
	7018	**7020**	**7022**	**7024**	**7026**	**7028**	**7030**	**2703**

T. G. C. set in this setion is
7017, 7018, 7019, 7020, 7021 and 7022.

8th set

	7982	7983	7984	7985	7986	7987	7988	7989
8	8000	8001	8002	8003	8004	8005	8006	8007
	8008	**8010**	**8012**	**8014**	**8016**	**8018**	**8020**	**8022**

8th set

7992	7993	7994	7995	7996	7997	7998	7999
8010	8011	8012	8013	8014	8015	8016	8017
8019	**8021**	**8023**	**8025**	**8027**	**8029**	**8031**	**8033**

T. G. C. set in this section is

8018, 8019, 8020, 8021, 8022 and 8023.

9th set

	8982	8983	8984	8985	8986	8987	8988	8989
9	9000	9001	9002	9003	9004	9005	9006	9007
	9009	**9011**	**9013**	**9015**	**9017**	**9019**	**9021**	**9023**
	8992	8993	8994	8995	8996	8997	8998	8999
	9010	9011	9012	9013	9014	9015	9016	9017
	9020	**9022**	**9024**	**9026**	**9028**	**9030**	**9032**	**9034**

The T. G. C. set of this section is

9019, 9020, 9021, 9022, 9023 and 9024.

Now if we follow the some crder to produce further set we do not get a next 2 generator number set. Thus if we observe the continuation of the law and prepare

9082	9083	9084
10000	10001	10002

etc. we do not get the same generated number from both.

Thus 9082 gives $9082 + 19 = 9101$ while 10000 gives 10001 and so we have reached the limit of our rule at set No. 9.

Up till now in 9 sets we gave in each set 16 numbers with 2 generators. Thus there are 144 numbers of 4 digits

each having 2 generators. These are all derived from the basic Co-generators. We kept *a* 982 to *a* 9989 and *a* 992 to a 999 as basic generators where *a* changes from 0 to 9 in the different sets.

The corresponding second set of numbers was (a + 1) 000 to (a + 1) 007 and (a + 10) 10 to (a + 10) 17 and we got the same number in the 3rd row which was our 2 generator numbers

These 144 numbers will be called as derived from basic cogenerators $\left.\begin{matrix} a\ 982 \\ (a+1)\ 000 \end{matrix}\right\}$ where in the first set a took value 0 and 0 + 1 respectively.

All these 144 numbers have 18 as the difference between their corresponding cogenerators and they have 4 digits. There is also another set of 2 generator numbers in which the digits are 4 and the difference between their consecutive cogenerators is 9.

I describe them now

Any of the 81 numbers described in chapter 1st can be made 4 digited by adding a 4th digit behind the 2 cogenerators.

Thus take for example set number 4 and 3 numbers only as $\frac{391\quad 392\quad 393}{400\quad 401\quad 402}$ now consider them as

and $\frac{0\ 391\quad 0\ 392\quad 0\ 393}{0\ 400\quad 0\ 401\quad 0\ 402}$

Now in place of 0 put any digit 1 to 9 and we get a corresponding new number of 4 digits from those two generators and this number is same from both the generation. Thus put for 0 value = 7 we get

7391 7392 7393

7400 7401 7402

7391 and 7400 both generate 7411

7392 and 7401 both generate 7413
and 7393 and 7402 both generate 7415

The table can be put as

7391	7392	7393
7400	7401	7402
7411	**7413**	**7315**

Thus the three numbers of 3 digits are made to form a new series of 3 numbers of 4 digits and they all have 2 generators.

As there are 1 to 9 digits we can put the value of zero by any of the value 1 to 9. Each of the set can prepare in this way 9 new sets and as there are 9 sets we can have $81 \times 9 = 729$ new numbers of 4 digits which have 2 generators and their cogenerators will differ by 9 only and not by 18.

N. B. — Thus in all these 729 numbers of 4 digits which have 2 generators and the cogenerators differ in them by 9.

Also there are 144 numbers of 4 digits which have 2 generators and whose cogenerators have difference of 18 in them.

Thus in short there are $729 + 144 = 873$ numbers of 4 digits which have 2 generators.

All other numbers of 4 digits are either self numbers or have only one generator for them. Just now I do not wish to count all the numbers of 1 generator end the self numbers in this range. That will form a separate branch of investigation.

However in order to have a correct understanding of this subject. I give instances to change some of the numbers in the 9 sets from 3 digits to 4 digits.

1st illustration — Let us change the first 9 numbers of the set No. 3 from 3 digits to 4 digits by putting 8 for 0

(regarding 0 as the first 4th digit as initial). Our steps will be following.

The first values of the set are

0291	0292	0293	0294	0295	0296	0297	0298	0299
0300	0301	0302	0303	0304	0305	0306	0307	0308

Changing the 0 to 8 we get the new generators as

8291	8292	8293	8294	8295	8296	8297	8298	8299
8300	8301	8302	8303	8304	8305	8306	8307	8308

Now these both generators give the same new 2 generated number and the new set of 4 digited numbers from them is as

8311 8313 8315 8317 8319 8321 8323 8325 8327

Illustration II

Let us change the last 4 numbers of set No. 8 by putting 5 as the value of zero.

The last 4 numbers of set No. 8 in last chapter are

0796	0797	0798	0799
0805	0806	0807	0808

changing 0 to 5 we get

5796	5797	5798	5799
5805	5806	5807	5808

The new corresponding 4 digited numbers having 2 generators will be as

5823 5825 5827 5829

Here the cogenerators differ by 9 only.

Many such illustrations can be taken to understand this subject.

CHAPTER III

Numbers with 3 generators

Introduction— Between 1 to 100 there are 13 self uumbers and 87 generated numbers having only 1 generator for them.

There is no number with 2 generators uptill 101. The first number with 2 generators is 101 and to the greatest number of 3 dig.ts having 2 generators is 925.

We have just now shown in last chapter that there are 873 numbers of 4 digits which have 2 generators. It is very difficult to count 2 generator numbers having 5 digits or 6 digits or 7 digits etc etc. It is very difficult to count and say exactly how many numbers of 4 digits or 5 digits or six digits have 1 generator how many have 2 generators and how many are self numbers. It is all an attempt just like trying to count the sand particle on a sea shore but in spite of all these curious counting remarks there is a certaion philosophy which tells us that there is no number with 3 generators between 1 to 1000,00,000,000,00 and the first number and the least number having 3 generators is 100,000,000,0000,1 or the least number having 3 numbers for its co-generators is ($10^{13}+1$) and the next number is $13^{13}+102$.

The first 2 numbers which have 3 generators are $10^{13}+1$ and $10^{13}+102$. How this is proved is a great philosuphy and it is to be noted that the first number in the series of 3 generator numbers has 14 digits. For the present believe it to be true. The three generators for $10^{13}+1$ will now be described.

The three generators for $10^{13} + 1$ are

I 1 000 000 000 0 000
or $1(0)_{10}$ 000

II 999 999 999 9 901
or $(9)_{10}$ 901

III 999 999 999 9 892
or $(9)_{10}$ 892

These give $10^{13} + 1$

This fact can be casily checked.

The next number of 3 generators has also 14 digits and it is $10^{13} + 102$.

The three generators for it are

I 1 000 000 000 0 100
$= 1(0)_{10}$ 100

II 1 000 000 000 0 091
$= 1(0)_{10}$ 091

III 999 999 999 9 992
$= (9)_{10}$ 992

All these numbers give the number

$1(0)_{10}$ 102

We write these numbers in the following type of short notation. The first 3 numbers in the column standing for generators and the 4th number for the generated number. The first 3 are the co-generators and the last is the 3 generator number that means this number has 3 generators as shown above.

(1)	(2)	(3)
$1(0)_{10}$ 000	$1(0)_{10}$ 100	$8(0)_{10}$ 106
$0(9)_{10}$ 901	$1(0)_{10}$ 091	$8(0)_{10}$ 097
$0(9)_{10}$ 892	$0(9)_{10}$ 992	$7(9)_{10}$ 998
$1(0)_{10}$ 001	**$1(0)_{10}$ 102**	**$8(0)_{10}$ 121**
(14 digits)	(14 digits)	(14 digits)

Now using this notation a table of several 3 generator number is given further. The first three lines are the number which are the generators of the last numbers which is shown in bold type, and the number of digits in it is put below in bracket.

A table of 3 generators numbers.

(4)	(5)	(6)
1 $(0)_{588}$ 5109	1 $(0)_{100}$ 00	1 $(0)_{342}$ 2980
$(9)_{588}$ 9808	$(9)_{100}$ 100	$(9)_{342}$ 9902
$(9)_{588}$ 9799	$(9)_{100}$ 091	$(9)_{342}$ 9893
1 $(0)_{588}$ 5125	**1 $(0)_{100}$ 001**	**1 $(0)_{342}$ 3000**
(593 digits)	(104 digits)	(347 digits)
(7)	(8)	(9)
1 $(0)_{133}$ 1006	1 $(0)_{578}$ 4100	1 $(0)_{80}$ 528
$(9)_{133}$ 9800	1 $(0)_{578}$ 4091	$(9)_{80}$ 808
$(9)_{133}$ 9791	$(9)_{578}$ 8880	$(9)_{80}$ 799
1 $(0)_{133}$ 1014	**1 $(0)_{578}$ 4106**	**1 $(0)_{80}$ 544**
(138 digits)	(583 digits)	(84 digits)
(10)	(11)	(12)
1 $(0)_{65}$ 002	1 $(0)_{25}$ 208	$(9)_{97}$ 819
$(9)_{65}$ 408	1 $(0)_{25}$ 199	1 $(0)_{97}$ 692
$(9)_{65}$ 399	$(9)_{25}$ 974	1 $(0)_{97}$ 701
1 $(0)_{65}$ 005	**1 $(0)_{25}$ 219**	**1 $(0)_{97}$ 710**
(69 digits)	(29 digits)	(101 digits)

In the last page 12 different examples are given where the final number in the 4th row is the Trigenerator. The number above it are the 3 generators for the number mentioned. The digits in the Trigenerator number are also mentioned.

The first three examples contain Trigenerators of 14 digits. In example 4 the Tri generator 1 $(0)_{588}$ 5125 has 593 digits and the numbers 1 $(0)_{588}$ 5109, $(9)_{588}$ 9808 and $(9)_{588}$ 9799 are the three numbers which are the cogenerators of that

Trigenerator number. The further example 5 to 12 contain trigenerators having 104, 347, 138, 583, 84, 69 and 29 digits in each of them respectively. Several examples can be constructed.

However it should be again noted that the least number having 3 generators is 1 000 000 000 000 1 ; These cannot be any number having less than 14 digits which has 3 generators. This fact is discovered on the assumtion that the least number of 2 generators is 101. and that there is no number less than 101 which has 2 generators can be very easily verified. (see also page 10 of the book Puzzles of self numbers).

CHAPTER IV

Numbers with four generators If N_1 is a number, d_1 the sum of the digits in N_1 then $N_1 + d_1 = N_2$ is the new number and is called the generated number from N_1. Conversely has every N_2 a corrosponding N_1 and d_1! If N_2 is such that $N_2 = N_1 + d_1$ is impossible then N_2 is the self number. If it possible that $N_2 = N_1 + d_1 = N'_1 + d'_1$ then the number has two generators. The first number of that type is 101.

If $N_2 = N_1 + d_1 = N'_1 + d'_1 = N''_1 + d''_1$ Then the N_2 has three generators and the first number of this type is $10^{13} + 1$ and 12 of that type are described.

A number bigger than this was first discovered on 2–11–58 at chromepet while discussing this subject seriously with Dr. A. Narasing Rao of Madras University and I have thanked him very much for it. After one year on 31–12–59 after various types of discussions with Professor Gunjikar of Bombay, a first number with four generators was found out. I am very much thankful to Prof. Gunjikar in the new ideas of that discussion. However this number having 4 generators was a very huge one. Further we both went on preparing such numbers with four generators. Our numbers were very big they had 140 digits in some of them or even biggers There was a competition between both of us for various months to discover numbers with less digits. At last on 7–6–61 the least number of 4 generators was discovered by me. It contains 25 digits. After about 4 months more Professor Gunjikar could also find the same number in his own methods of procedure. Now both of us come to same conclusion that the least number of 4 generators has 25 digits and it is as

$$= 1\,(0)_{21}\,102$$

These : $\left.\begin{array}{r} (9)_{21}\,893 \\ (9)_{21}\,902 \\ 1\,(0)_{21}\,091 \\ 1\,(0)_{21}\,100 \end{array}\right\}$ give $\mathbf{1\,(0)_{21}\,102}$

This number will have a historical value in future as discovered by D. R. Kaprekar on 7–6–61 by his methods and also discovered by Prof. Gunjikar on 7–9–61 by his different methods.

Now further is given a table of various types of four generator numbers.

It must be noted that it is very easy to prepare huge numbers of such type that they have four generators but it is difficult to prepare numbers having smaller number of digits in them. The papers on construction of such number were send by me first at Bharat Ganit Parishad at LUCKNOW in 1960 and further at 2 more conferences at Bombay and Ahamadabad.

Now here is the table of various 4 generator numbers.

The number of digits in each of them are also given below, and each result should be checked by the reader.

(1)	(2)	(3) (same as 2nd)
$(9)_{100}\,891$	$(9)_{356}\,891$	$(9)_{355}\,9891$
$(9)_{100}\,900$	$(9)_{356}\,900$	$(9)_{355}\,9900$
$1\,(0)_{100}\,800$	$1\,(0)_{355}\,3104$	$1\,(0)_{355}\,3104$
$1\,(0)_{100}\,791$	$1\,(0)_{355}\,3095$	$1\,(0)_{355}\,3095$
$\mathbf{1\,(0)_{100}\,809}$	$\mathbf{1\,(0)_{355}\,3113}$	$\mathbf{1\,(0)_{355}\,3113}$
(104 digits)	(360 digits)	(360 digits)

(4)	(6)	(6)
$(9)_{1111}$ 1105	$(9)_{588}$ 9791	$(9)_{155}$ 9794
$(9)_{1111}$ 1096	$(9)_{588}$ 9800	$(9)_{155}$ 9803
$1(0)_{1111}$ 1104	$1(0)_{588}$ 5101	$1(0)_{155}$ 1198
$1(0)_{1111}$ 1095	$1(0)_{588}$ 5092	$1(0)_{155}$ 1207
$1(0)_{1111}$ 1111	**$1(0)_{588}$ 5109**	**$1(0)_{155}$ 1218**
(1116 digits)	(593 digits)	(160 digits)
(7)	(8)	(9)
$(9)_{133}$ 9892	$0(9)_{133}$ 9792	$(9)_{100}$ 191
$(9)_{133}$ 9901	$0(9)_{133}$ 9801	$(9)_{100}$ 200
$1(0)_{133}$ 1098	$1(0)_{133}$ 0989	$1(0)_{100}$ 091
$1(0)_{133}$ 1107	$1(0)_{133}$ 1007	$1(0)_{100}$ 100
$1(0)_{133}$ 1117	**$1(0)_{133}$ 1016**	**$1(0)_{100}$ 102**
—138 digits—	—138 digits—	—104 digits—
(10)	(11)	(12)
$(9)_{1111}$ 1107	$(9)_{133}$ 9900	$(9)_{1111}$ 3107
$(9)_{1111}$ 1098	$(9)_{133}$ 9891	$(9)_{1111}$ 3098
$1(0)_{1111}$ 1106	$1(0)_{133}$ 1106	$1(0)_{1111}$ 3097
$1(0)_{1111}$ 1097	$1(0)_{133}$ 1097	$1(0)_{1111}$ 3106
$1(0)_{1111}$ 1115	**$1(0)_{133}$ 1115**	**$1(0)_{1111}$ 3117**
—1116 digits—	—138 digits—	—1116 digits—
(13)	(14)	(15)
$1(0)_{1001}$ 407	$1(0)_{900}$ 0398	$(9)_{100}$ 794
$1(0)_{1001}$ 398	$1(0)_{900}$ 0407	$(9)_{100}$ 803
$(9)_{1000}$ 1398	$(9)_{900}$ 2307	$1(0)_{100}$ 694
$(9)_{1000}$ 1407	$(9)_{900}$ 2298	$1(0)_{100}$ 703
$1(0)_{1001}$ 419	**$1(0)_{900}$ 0419**	**$1(0)_{100}$ 714**
—1005 digits—	—905 digits—	—104 digits—
(16)	(17)	(18)
$(9)_{111}$ 192	$(9)_{43}$ 705	$(9)_{21}$ 894
$(9)_{111}$ 201	$(9)_{43}$ 696	$(9)_{21}$ 903
$1(0)_{111}$ 191	$1(0)_{43}$ 092	$1(0)_{21}$ 092
$1(0)_{111}$ 200	$1(0)_{43}$ 101	$1(0)_{21}$ 101
$1(0)_{111}$ 203	**$1(0)_{43}$ 104**	**$1(0)_{21}$ 104**
—115 digits—	—47 digits—	—25 digits—

(19)

$(9)_{123}9891$
$(9)_{123}9900$
$1(0)_{123}1016$
$1(0)_{123}0998$
$\mathbf{1(0)_{123}1025}$
—128 digits—

(20)

$(9)_{111}408$
$(9)_{111}399$
$1(0)_{111}407$
$1(0)_{111}398$
$\mathbf{1(0)_{111}419}$
—115 digits—

(21)

$(9)_{56}891$
$(9)_{56}900$
$1(0)_{56}404$
$1(0)_{56}395$
$\mathbf{1(0)_{56}413}$
—60 digits—

(22)

$(9)_{23}891$
$(9)_{23}900$
$1(0)_{23}098$
$1(0)_{23}107$
$\mathbf{1(0)_{23}116}$
—27 digits—

(23)

$(9)_{111}892$
$(9)_{111}901$
$1(0)_{111}891$
$1(0)_{111}900$
$\mathbf{1(0)_{111}910}$
—115 digits—

(24)

$(9)_{111}799$
$(9)_{111}808$
$1(0)_{111}798$
$1(0)_{111}807$
$\mathbf{1(0)_{111}823}$
—115 digits—

(25)

$(9)_{34}902$
$(9)_{34}893$
$1(0)_{34}208$
$1(0)_{34}199$
$\mathbf{1(0)_{34}219}$
— 38 digits

(25)

$(9)_{23}892$
$(9)_{23}901$
$1(0)_{23}108$
$1(0)_{23}099$
$\mathbf{1(0)_{23}118}$
— 27 digits

(27)

$(9)_{45}693$
$(9)_{45}702$
$1(0)_{45}107$
$1(0)_{45}098$
$\mathbf{1(0)_{45}116}$
— 49 digits

(28)

$(9)_{21}899$
$(9)_{21}908$
$1(0)_{21}106$
$1(0)_{21}097$
$\mathbf{1(0)_{21}114}$
— 25 digits

(29)

$(9)_{78}800$
$(9)_{78}791$
$1(0)_{78}493$
$1(0)_{78}502$
$\mathbf{1(0)_{78}510}$
— 82 digits

(30)

$(9)_{21}99891$
$(9)_{21}99900$
$1(0)_{21}00098$
$1(0)_{21}00107$
$\mathbf{1(0)_{21}00116}$
— 27 digits

Up till now 30 different types of 4 generators numbers are described.

From any number of 4 generator we can find within certain limits other numbers of the 4 generator type. We are to go on changing the digits in one of the generator and make corresponding changes in the other sets of digits in other generators also and find the new number generated.

This change should but be made only in such a way that 9 in any of the position is not compelled to be made 10.

The new numbers can be prepared either by increasing the digits or by decreasing the digits. The least number of the series is called the primitive solution.

From any primitive solution we can generally get about 8 new solutions if the digits in the generators are small. However this all depends on the value of digit in one generator and corresponding digits in the other generators so that 9 is not changed to 10 anywhere.

The various methods of preparing such numbers will be described in my next publication. Also numbers of 5, 6, 7 and 8 generators will be described in next publication.

Further is now given a list of peculiar self numbers self Demlo numbers and self primes.

I hope the reader will take great pleasure in the joy felt in preparation of all sorts of numbers in this book and I request the reader to inform me all his suggestions and any mistakes that he will be able to find in my work. Hope reader will find great joy in this book.

This is really enjoying various types of puzzles from big sized numbers. It is an evergreen type of joy in mathematics.

311, Devlali Camp
8-5-1962.

D. R. KAPREKAR

APPENDIX

Peculiar self numbers.

A list of self numbers of some peculiar structures is given below. They are proved to be self by some independent method.

(1) Million or 1000000 is a self number see my book puzzles of self numbers. Page 18.

(2) 31, 53, 97, 211, 389, 569, 727, 883, and 1109 are self primes.

(3) 121, 222, 132, 143, 154, 165, 176, 187, and 198 are self Demlo numbers.

(4) The famous $\frac{1}{7} = 142857$ has 142857 a self number.

(5) Concentric self numbers are such that the sum of the middle 2 digits is equal to the sum of the extreme digits. Thus 1278 has $1 + 8 = 9$ and $2 + 7 = 9$. It is a concentric self number.

The following is a list of concentric self number.

(a) 1021, 1032, 1043, 1054, 1065, 1078, 1087, 1098.

(b) 1111, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199.

(c) 1212, 1223, 1234, 1245, 1256, 1267, 1278, 1289.

N. B.—1234 is concentric self number with digits in increasing order.

400

(6) 64 and ~~1~~00 are self square numbers.

(7) 3, and $(3)_{10}$ = 333 333 333 3 are self.

(8) After 10^6 = million the next is 10^{16} as a power of 10 which is a self number.

$$\left.\begin{array}{ll} (9) & 9999\ 99999\ 00 \\ \text{and} & 9999\ 99999\ \ 2 \end{array}\right\}\ \text{are self numbers.}$$

note here after 9 written 9 times we can write further either 2 zeroes or simply 2 and both are self.

(10) $100002 = 1\,(0)_4\,2$

and $1000000000000002 = 1\,(0)_{14}\,2$

These both are self.

(11) 108 is a self number and if instead of one zero we put twelve zeroes then also it is self. That means 1000 000 000 0008 is also self. The above is a collection from my heap of note books where I have made lot of calculations and noted well the self numbers whenever I have found them.

(12) 111, 111, 111, 111, 111, 11 $= (1)_{17}$ is a self number. That means 1 written seventeen times is a self number.

— This has much importance —

Several more important results wiil be described in my next Volume.

POINTS FOR SILENTLY THINKING

(1) To find a formula for a self number. (up till now there is no formula for a prime number. Same is the case of the self number)

(2) Given any number N To find the nearest self number and the nearest junction number.

For example: N = 1339. For this the nearest junction numbers are 1520 and 1318, The nearest self number is 1109

(3) To find the sum of n terms of a digitadition series.

(4) To find the law in difference of twin self numbers. Thus in the beginning pairs 108, 110,2 09, 211 etc. differ by 2. Further this difference goes to 15; To find the next difference.

(5) To find sets of consecutive numbers which have 2 generators where each set contains more than 5 numbers for five number sets see pages 5 to 9.

(6) To prove that, number of self numbers is infinite.

(7) To find a formula for a junction number with 2, 3, or 4 junctions.

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